Unit 3 Conics

Unit 3 Day #1 – Completing the Square

Objective – ① You will solve Quadratic equations by completing the square.

Completing the square – A method to solve a quadratic equation by hand when factoring is difficult or not possible.

Procedure:

1) Add/Subtract c to the other side of the equation.
2) Divide both sides by a
3) Add $\left(\frac{b}{2}\right)^2$ to both sides
4) Factor the left side, add the right side
5) Take the square root of both sides
6) Solve for x.

Example #1 Find the roots of $x^2-4x-96=0$ using completing the square.

$$x^2-4x-96=0 \longrightarrow x^2-4x+\boxed{(4/2)^2}$$

$$x^2-4x+4=96+4$$

$$x^2-4x+4=100$$

$$\sqrt{(x-2)^2}=\sqrt{100}$$

$$x-2=\pm 10$$

$$x=\pm 10+2$$

$$\boxed{x=12} \quad \boxed{x=-8}$$

Example #2

Find the roots of $x^2 - 5x + 11 = 0$ using completing the square

$$x^2 - 5x + 11 = 0$$

$$x^2 - 5x + \left(\tfrac{1}{2}(-5)\right)^2 = -11 + \left(\tfrac{1}{2}(-5)\right)^2$$

$$x^2 - 5x + \tfrac{25}{4} = -11 + \left(\tfrac{25}{4}\right)$$

$$\sqrt{(x - 5/2)^2} = \sqrt{-4.75}$$

$$x - 5/2 = \sqrt{4.75}\,\sqrt{-1}$$

$$x - 5/2 = \pm i\sqrt{\tfrac{19}{4}}$$

$$\boxed{x = \tfrac{5}{2} + i\sqrt{\tfrac{19}{4}}} \qquad \boxed{x = \tfrac{5}{2} - i\sqrt{\tfrac{19}{4}}}$$

Example #3: Find the roots of $2x^2 + 5x - 9 = 0$ using completing the square

$$2x^2 + 5x - 9 = 0$$

$$\frac{2x^2}{2} + \frac{5x}{2} + \quad = \frac{9}{2} +$$

$$x^2 + \tfrac{5}{2}x + \left(\tfrac{1}{2}(5/2)\right)^2 = \tfrac{9}{2} + \left(\tfrac{1}{2}(5/2)\right)^2$$

$$x^2 + \tfrac{5}{2}x + \tfrac{25}{16} = \tfrac{9}{2} + \tfrac{25}{16}$$

$$\sqrt{\left(x + \tfrac{5}{4}\right)^2} = \sqrt{\tfrac{97}{16}}$$

$$x + \tfrac{5}{4} = \frac{\pm\sqrt{97}}{4}$$

$$-\tfrac{5}{4} \qquad -5/4$$

$$x = -\frac{5}{4} + \frac{\sqrt{97}}{4} \qquad x = -\frac{5}{4} - \frac{\sqrt{97}}{4}$$

HW: Solve by completing the square

1) $x^2 - 10x + 121 = 0$ 2) $x^2 - 3x - 7 = 0$ 3) $3x^2 - 12x = -4$

Unit 3 Day 2 → Circles

Objective - You will use and determine the standard and general forms of the equation of a circle.

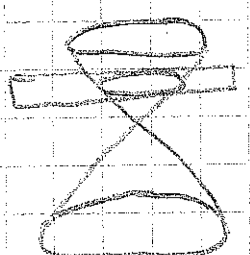
Circles:

Standard Form: $(x-h)^2 + (y-k)^2 = r^2$

center: (h, k)

radius: r

General form: $x^2 + y^2 + Dx + Ey + F = 0$



Converting from General Form to Standard Form.

1) Group together the x's, group together the y's, and move the constant to the other side of the equation.

2) Complete the square for the x terms and complete the squares for the y terms.

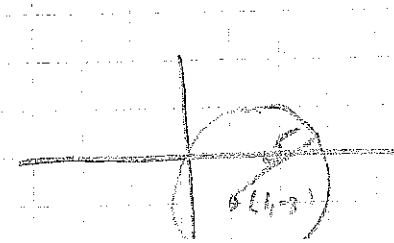
3) Rewrite the left side of the equation as the sum of two binomials squared.

Example #1: Write the stand form of $x^2+y^2-2x+6y-15=0$ and sketch

$$x^2+y^2-2x+6y-15=0$$

$$x^2-2x+\boxed{1}+y^2+6y+\boxed{9} = 15+\boxed{1}+\boxed{9}$$

$$(x-1)^2+(y+3)^2 = 25$$



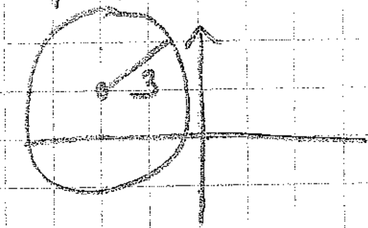
$16x^2+16y^2+64x-32y-64=0$

$16(x^2+y^2+4x-2y-4)=0$

$16(\underbrace{x^2+4x+\boxed{4}}+\underbrace{y^2-2y+\boxed{1}}) = 4+\boxed{4}+\boxed{1}$

$(x+2)^2+(y-1)^2=9$

(-2, 1) r=3



★ Example #3: Write the standard form of the equation of a circle that passes through (5,3) (-2,2) and (-1,-5)

(x, y) $x^2+y^2+Dx+Ey+F=0$

(5,3) $5^2+3^2+5D+3E+F=0$

$25+9+5D+3E+F=0$

$\boxed{5D+3E+F+34=0}$

(-2,2) $(-2)^2+(2)^2+-2D+2E+F=0$

$\boxed{-2D+2E+F+8=0}$

(-1,-5) $(-1)^2+(-5)^2-D-5E+F=0$

$\boxed{-1D-5E+F+26=0}$

$5D+3E+F=-34$

$-2D+2E+F=-8$

$-1D-5E+F=-26$

$-2D+2E+F=-8$

$2D+10E-2F=52$

$12E-F=44$

$10D+6E+2F=-68$

$-10D+10E+5F=-40$

$16E+7F=-108$

$16E+7F=-108$

$7(12E-F=44)$

$16E+7F=-108$

$84E-7F=308$

$100E=200$

$\boxed{E=2}$

$16(2)+7F=-108$

$32+7F=-108$

$\boxed{F=-20}$

$5D+3(2)-20=-34$

$5D-14=-34$

$\boxed{D=-4}$

$x^2+y^2-4x+2y-20=0$

$\underbrace{x^2-4x+\boxed{4}}+\underbrace{y^2+2y+\boxed{1}} = 20+\boxed{4}+\boxed{1}$

$(x-2)^2+(y+1)^2=25$

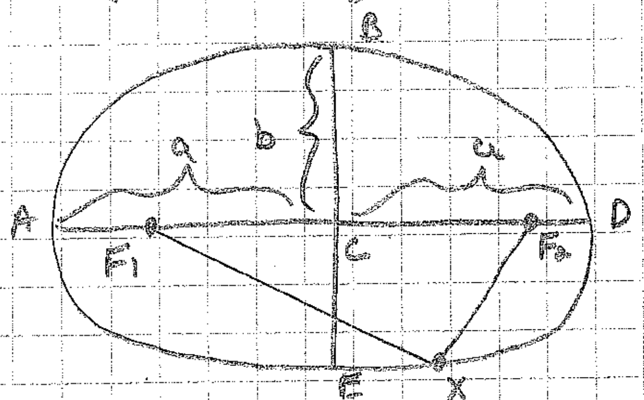
Center: (2, -1) r=5

HW: Pg 628 # 23, 25

Unit 3 Day #3 Ellipses

Objective - You will ① Use + Determine the standard and general forms of the equation of an ellipse
② Graph ellipses

Ellipse - the set of all points in the plane, the sum of whose distances from 2 fixed points (called foci) is constant.



AD: major axis
AC = DC : semi- major axis
BE: minor axis
BC = CE : semi-minor axis

Vertices: A, B, D, E

Center: C

Foci: F_1, F_2

Ellipse Formulas

Standard Form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

The denominator with the large number shows the direction in which the ellipse will be graphed.

Eccentricity - describes the shape of the ellipse.

$$e = \frac{c}{a} = \frac{\text{distance from center to focus}}{\text{distance from center to vertex of major axis}}$$

$$c = \sqrt{a^2 - b^2}$$

Example #1

Given: $\frac{(y-3)^2}{25} + \frac{(x+4)^2}{9} = 1$

5, 3

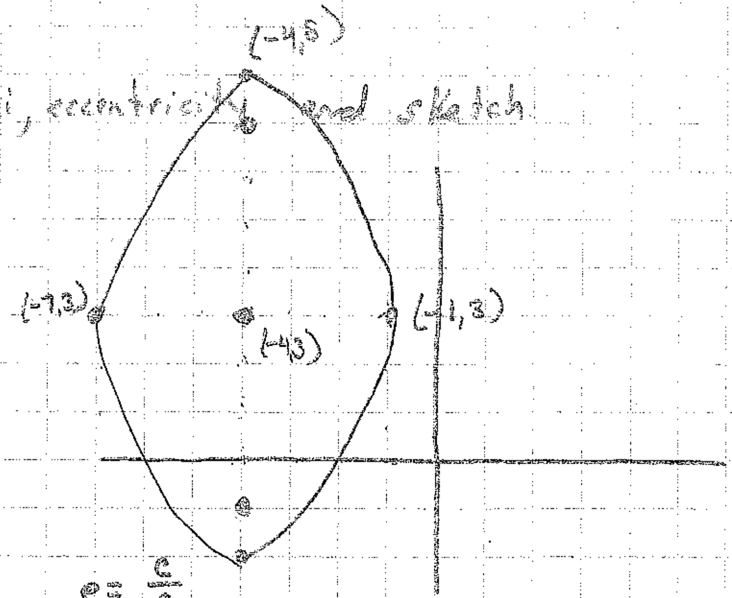
Find center, vertices, foci, eccentricity and sketch

Center: $(-4,3)$

Vertices: $(-7,3)\ (-1,3)$
$(-4,8)\ (-4,-2)$

Foci: $c = \sqrt{a^2 - b^2}$
$c = \sqrt{25-9}$
$c = \sqrt{16}$
$\boxed{c = 4}$

$(-4,-1)$
$(-4,7)$



$e = \frac{c}{a}$

$e = \boxed{\frac{4}{5}}$

Example #2

Given: $4x^2 + 9y^2 + 40x + 36y + 100 = 0$

Find center, vertices, foci, eccentricity & sketch

$(x-h)^2$
$4x^2 - 40x$
$4(x^2 - 10x)$
$4(x^2 - 10x + 25)$
$(\frac{-10}{2})^2$
$= 25$

$4x^2 - 40x + 9y^2 + 36y = -100$

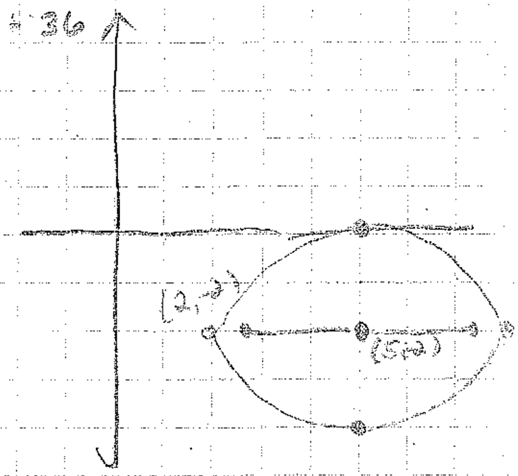
$4(x^2 - 10x + 25) + 9(y^2 + 4y + 4) = -100 + 100 + 36$

$\frac{4(x-5)^2}{36} + \frac{9(y+2)^2}{36} = \frac{36}{36}$

$3 \quad \frac{(x-5)^2}{9} + \frac{(y+2)^2}{4} = 1 \quad 2$

$(y-k)^2$
$9y^2 + 36y$
$9(y^2 + 4y)$
$9(y^2 + 4y + 4)$
$(\frac{4}{2})^2$
$= 4$

center: $(5,-2)$



Vertices: $(2,-2)\ (8,-2)$
$(5,0)\ (5,-4)$

foci: $c = \sqrt{9-4}$
$c = \sqrt{5}$

$e = \frac{c}{a}$

$\boxed{e = \frac{\sqrt{5}}{3}}$

HW Pg 638
21, 24

Unit 3 Day #4 Ellipses (2nd Day)

Writing the Equation of an Ellipse

Recall:

Standard Form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

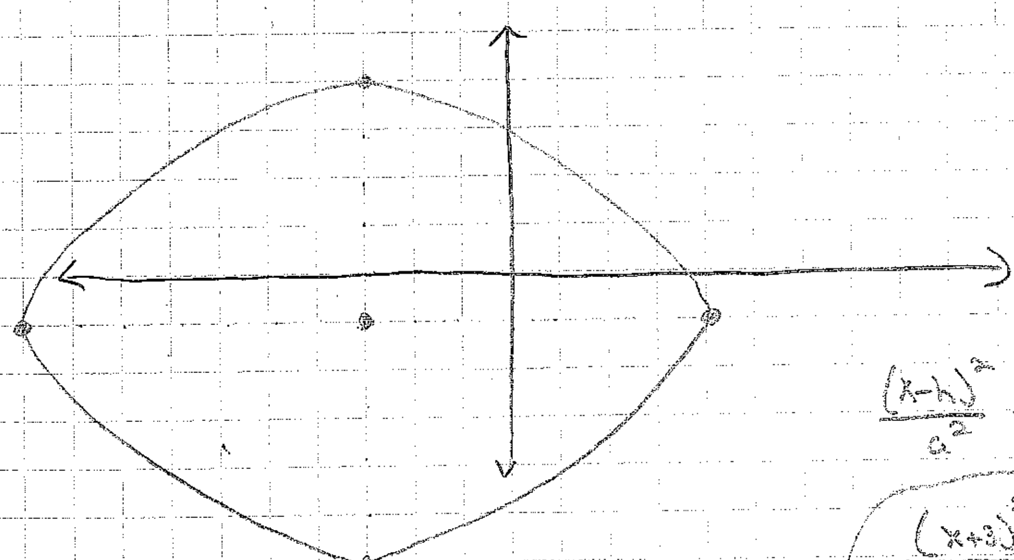
$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

Eccentricity - describes the shape of the ellipse

$e = \frac{c}{a} = \frac{\text{distance from center to focus}}{\text{distance from center to vertex of major axis}}$

$c = \sqrt{a^2 - b^2}$

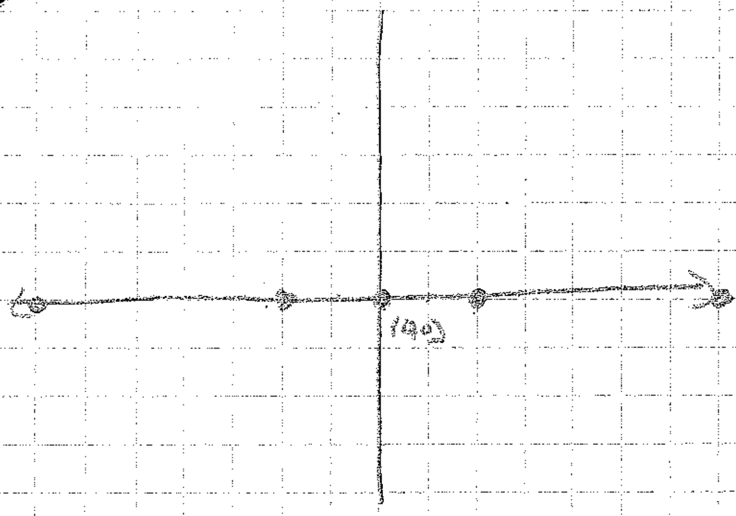
Example #1 Write the equation of the ellipse that has a center of (-3,-1), a horizontal semi-major axis of 7, and a semi-minor axis of 5.



$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2}$

$\frac{(x+3)^2}{49} + \frac{(y+1)^2}{25} = 1$

Example #2 - Write the equation of the ellipse that has foci of $(-2,0)$ and $(2,0)$, and $a=7$.



$$\frac{x^2}{49}+\frac{y^2}{45}=1$$

$a=7 \quad c=2$

$c=\sqrt{a^2-b^2}$

$(2)^2=(\sqrt{49-b^2})^2$

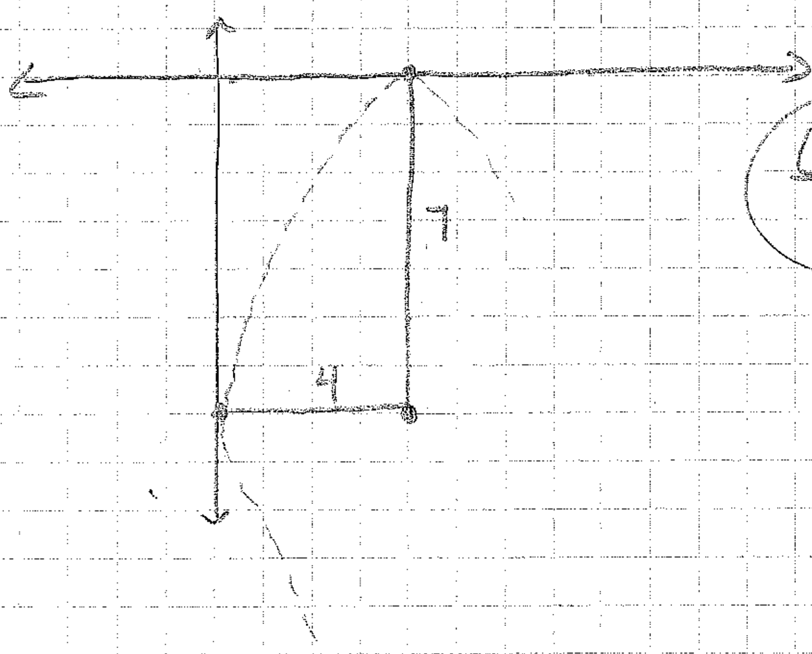
$4=49-b^2$

$-49 \quad -49$

$-45=-b^2$

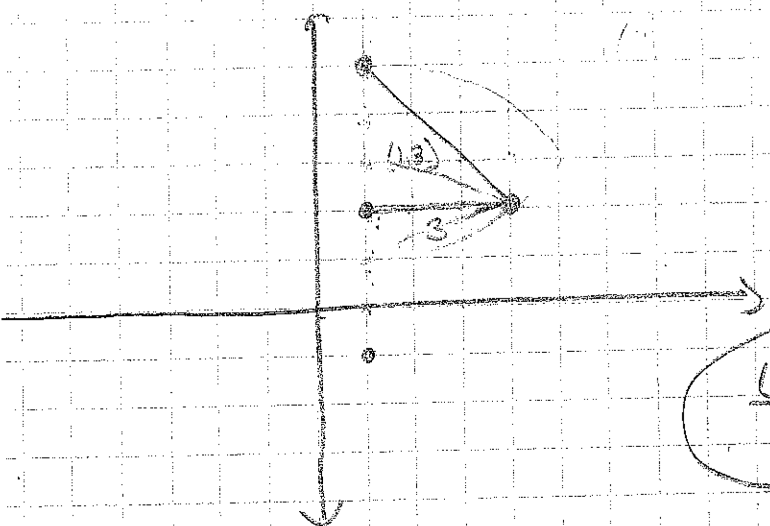
$\sqrt{b^2}=\sqrt{45}$

Example #3: Write the equation of the ellipse that is tangent to the x and y axes and has a center at $(4,-7)$.



$$\frac{(x-4)^2}{16}+\frac{(y+7)^2}{49}=1$$

Example #4 – Write the equation of the ellipse that has foci of $(1,-1)$ and $(1,5)$, and passes through the point $(4,2)$.



$c=3$

center: $(1,2)$

$b=3$

$b^2=9$

$c=\sqrt{a^2-b^2}$

$(3)^2=\left(\sqrt{a^2-9}\right)^2$

$9=a^2-9$

$+9 \quad +9$

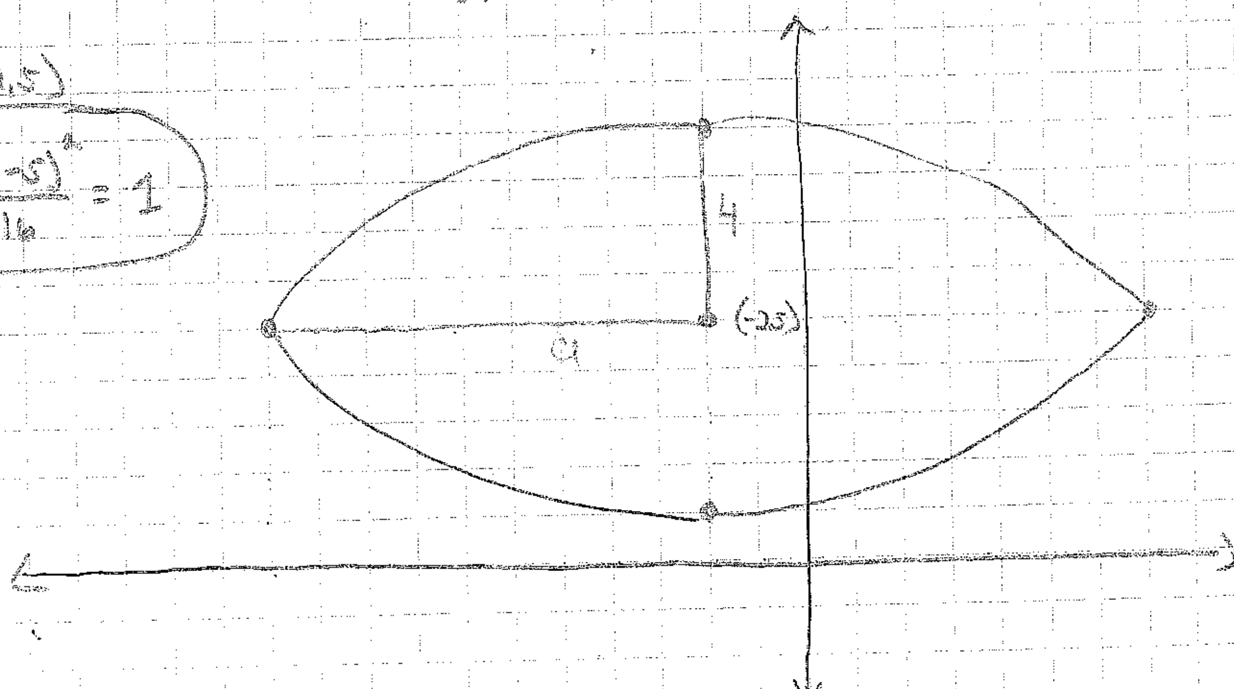
$18=a^2$

$$\frac{(x-1)^2}{\sqrt{18}}+\frac{(y-2)^2}{9}=1$$

Example #5 – Write the equation of the ellipse that has vertices of $(-11,5)$, $(7,5)$, $(-2,9)$ and $(-2,1)$.

center: $(-2,5)$

$$\frac{(x+2)^2}{81}+\frac{(y-5)^2}{16}=1$$



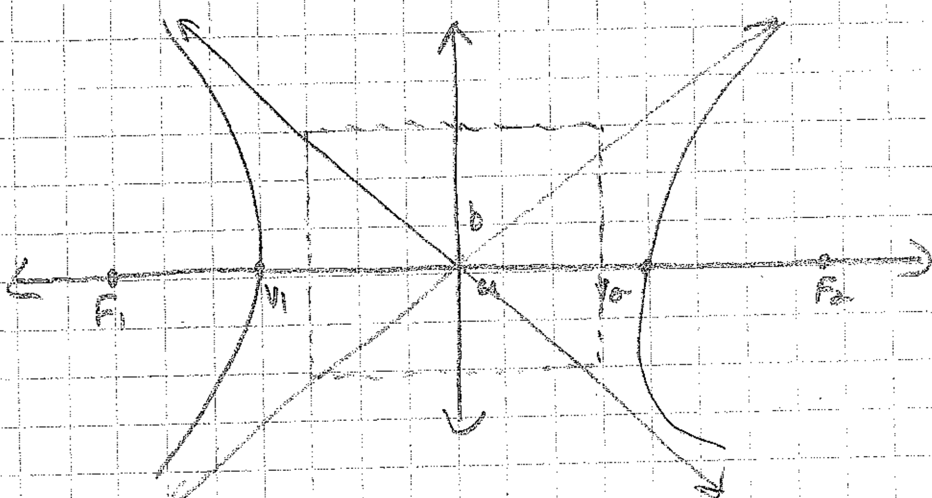
HW: Find the center, vertices, foci and eccentricity of:

$$6x^2+4y^2+24x-32y+64=0$$

Unit 3 Day 5 – Hyperbolas

Objective – ① Use + Determine the standard & general forms of the equation of a hyperbola.
② Graph hyperbolas.

Hyperbola – The set of all points in the plane in which the difference of the distances from two distinct fixed points, called foci, is constant.



Vertices: V_1, V_2
Foci: F_1, F_2
Transverse Axis (major axis): $2a$
Conjugate Axis (minor axis): $2b$

Hyperbola Formulas:

x first so....

Standard Form: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

y first so....

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Distance from center to focus: c

$c = \sqrt{a^2+b^2}$

Eccentricity: $e = \frac{c}{a}$

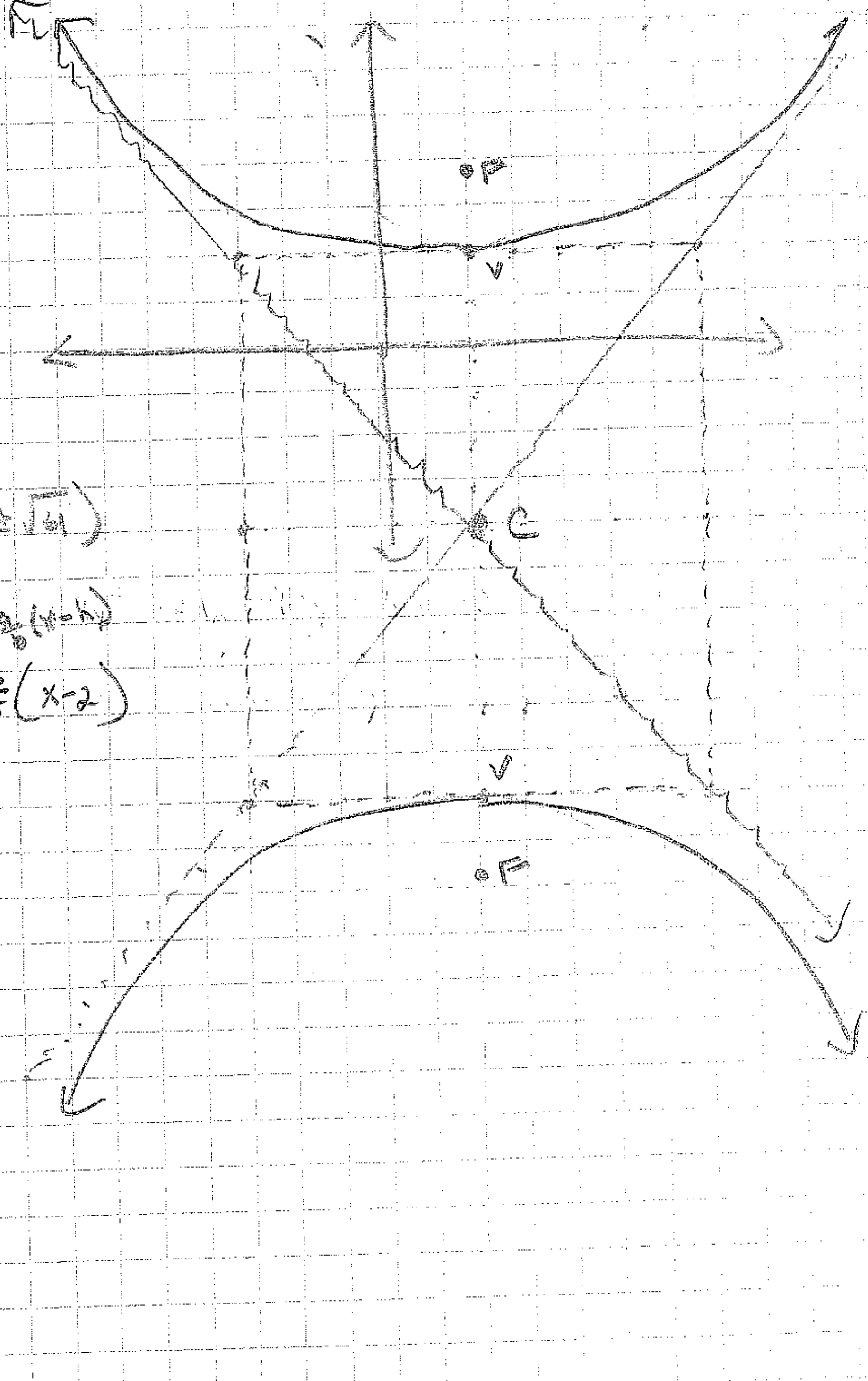
Equations of Asymptotes:

$y-k = \pm \frac{b}{a}(x-h)$

Example #1

Given: $\frac{(y+4)^2}{36} - \frac{(x-2)^2}{25} = 1$

Find the center, vertices, foci, eccentricity, asymptotes and sketch.



center: $(2,-4)$

Vertices: $(2,2)$ $(2,-10)$

foci: $a^2=36$ $\quad b^2=25$

$a=6$ $\quad b=5$

$c=\sqrt{a^2+b^2}$

$c=\sqrt{36+25}$

$c=\sqrt{61}$ $\quad (2, -4 \pm \sqrt{61})$

$e = \frac{c}{a} = \boxed{\frac{\sqrt{61}}{6}}$

$y-k = \pm \frac{a}{b}(x-h)$

asymptotes: $y+4 = \pm \frac{6}{5}(x-2)$

HW: Pg 650 # 17, 21

Example #2   Given: $-4x^2 + 9y^2 - 24x - 90y + 153 = 0$

Find the center, vertices, foci, eccentricity, asymptotes and sketch.

$$-4x^2 - 24x + 9y^2 - 90y = -153$$
$$-4(x^2 + 6x + 9) + 9(y^2 - 10y + 25) = -153 + \boxed{-36} + \boxed{225}$$
$$\frac{-4(x+3)^2}{-9\,36} + \frac{9(y-5)^2}{4\,36} = \frac{36}{36}$$
$$\frac{(y-5)^2}{4} - \frac{(x+3)^2}{9} = 1$$

$(x-h)^2$:
$-4x^2 - 24x$
$-4(x^2 + 6x + \boxed{9})$

$(y-k)^2$:
$9y^2 - 90y$
$9(y^2 - 10y + \boxed{25})$

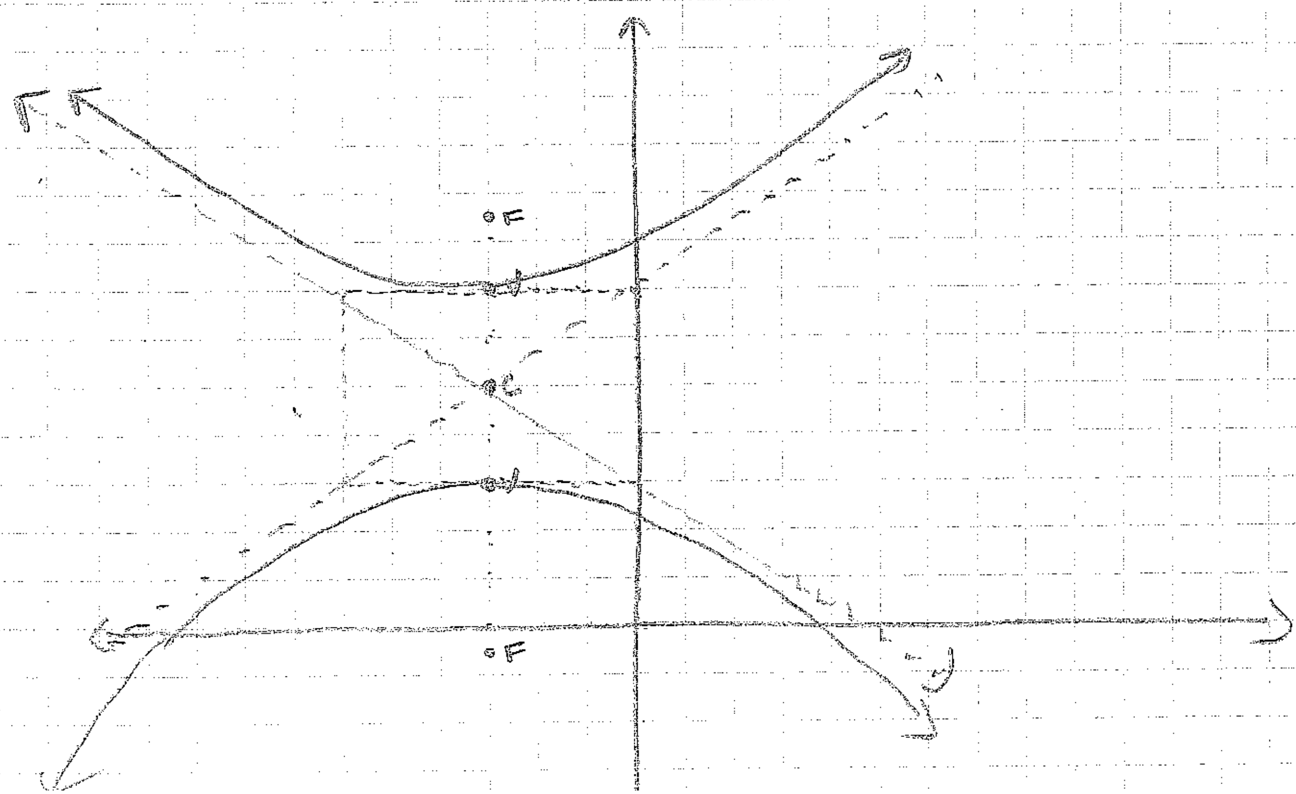
Center: $(-3, 5)$

Vertices: $(-3, 3)$ $(-3, 7)$

Foci: $c = \sqrt{a^2 + b^2}$ $(-3, 5 \pm \sqrt{13})$
$c = \sqrt{4 + 9}$
$c = \sqrt{13}$

Asymptotes: $y - 5 = \pm \frac{2}{3}(x + 3)$

$e = \frac{c}{a}$
$$\boxed{e = \frac{\sqrt{13}}{2}}$$



H.W. Pg 610

Day # 6 - Hyperbolas (con't) (Day #2)

Recall:

Hyperbola Formulas

Standard Form: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Distance from center to focus: c

$$c = \sqrt{a^2 + b^2}$$

Eccentricity: $e = \frac{c}{a}$

Equations of Asymptotes: $y - k = \pm\frac{b}{a}(x-h)$

$$y - k = \pm\frac{a}{b}(x-h)$$

Write the equation of the hyperbola given the following:

1) Foci: (0,8) (0,-8)

Eccentricity: $4/3$

$c = 4$

$a = 3$ $\boxed{a^2 = 9}$

Center: (0,0)

$c = \sqrt{a^2 + b^2}$

$(4)^2 = \left(\sqrt{9 + b^2}\right)^2$

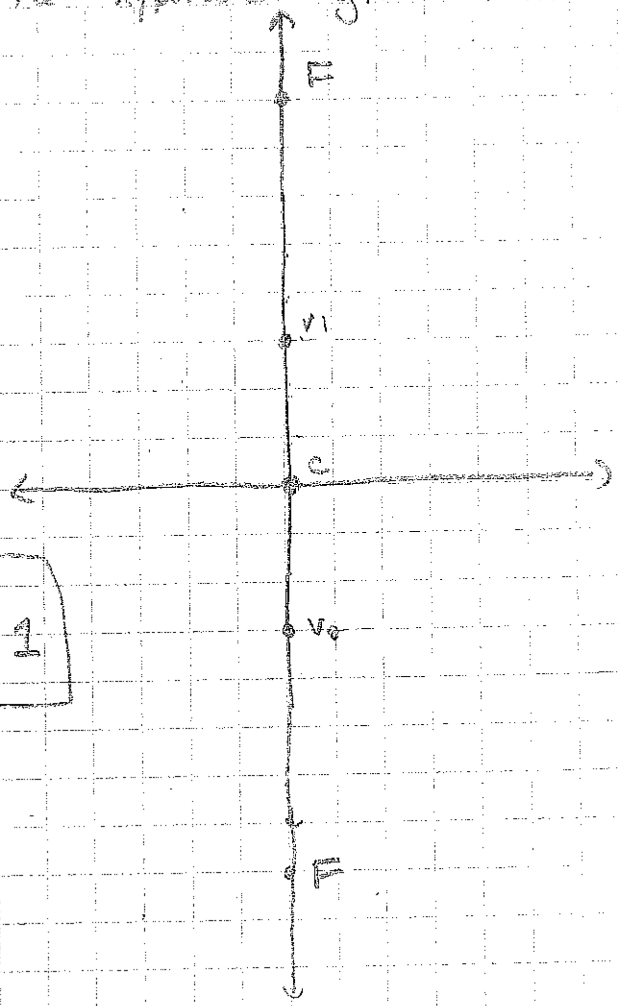
$16 = 9 + b^2$

$-9 \quad -9$

$\boxed{7 = b^2}$

$$\boxed{\frac{y^2}{9} - \frac{x^2}{7} = 1}$$

$c = \sqrt{16}$



2) Foci (7,1) (-3,1)

Transverse (major) axis: 8

Center: (2,1)

$a = 4$ $a^2 = 16$

$c = 5$

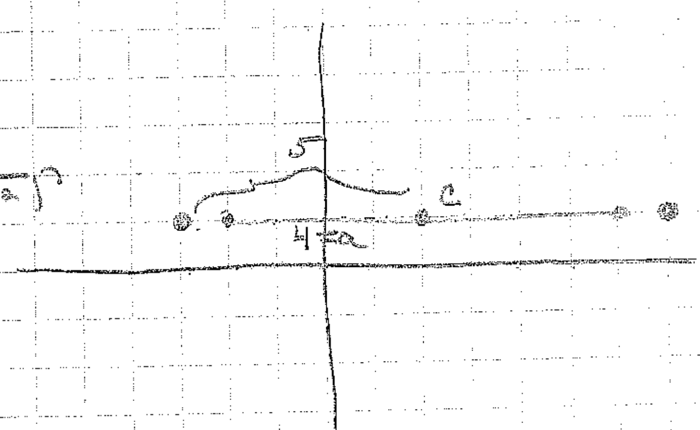
$(5)^2 = \left(\sqrt{16 + b^2}\right)^2$

$25 = 16 + b^2$

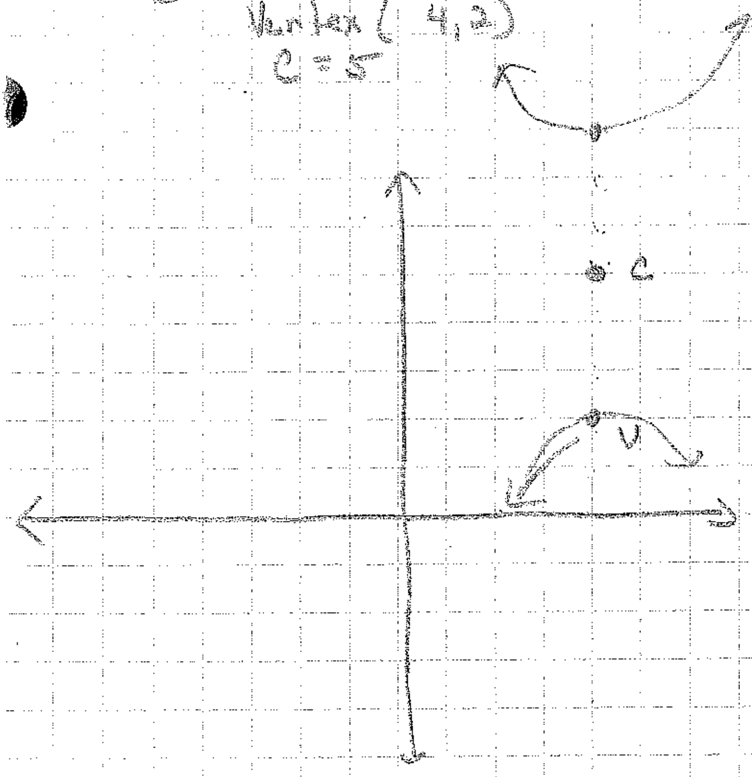
$-16 \quad -16$

$9 = b^2$

$$\boxed{\frac{(x-2)^2}{16} - \frac{(y-1)^2}{9} = 1}$$



3) Center $(-4, 5)$
Vertex $(-4, 2)$
$c = 5$



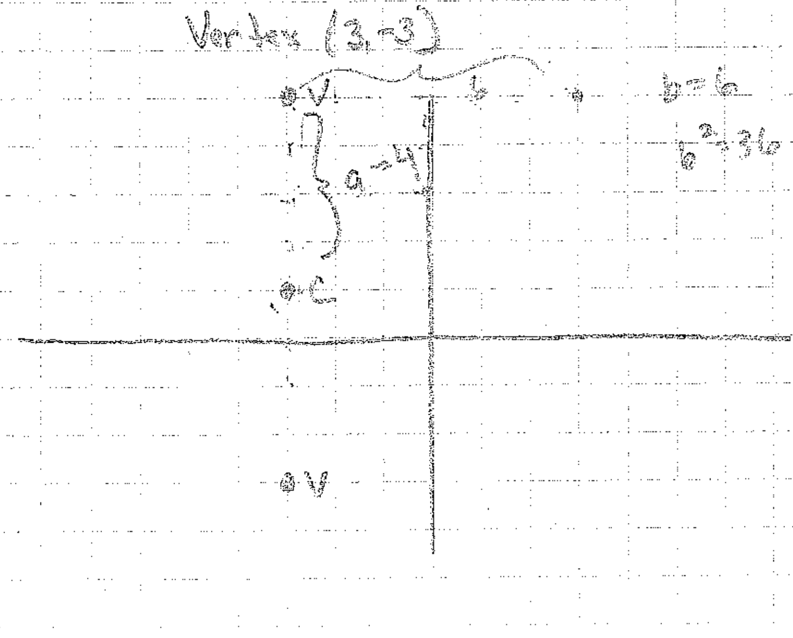
$a^2 = 9$
$a = 3$

$$\boxed{\frac{(y-5)^2}{9} - \frac{(x+4)^2}{16} = 1}$$

$c = \sqrt{a^2+b^2}$
$(5 = \sqrt{9+b^2})^2$
$25 = 9 + b^2$
$-9 \quad -9$
$\boxed{16 = b^2}$

4) Asymptotes: $y-1 = \pm\frac{2}{3}(x+3)$

Vertex $(3, -3)$



$b = 6$
$b^2 = 36$

Center: $(-3, 1)$

$$\boxed{\frac{(y-1)^2}{16} - \frac{(x+3)^2}{36} = 1}$$

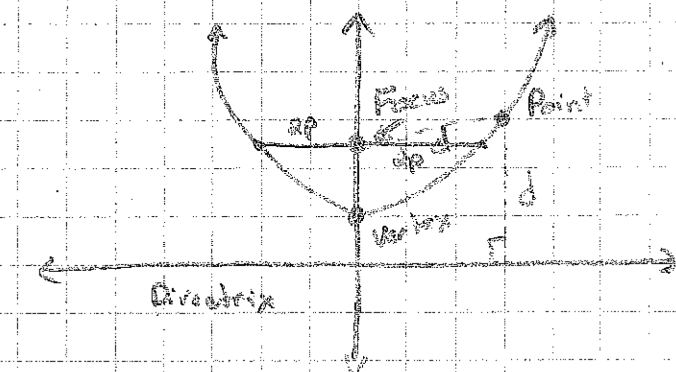
HW: Find the center, vertices, foci, eccentricity and sketch

$$\frac{(y+2)^2}{16} - \frac{(x+3)^2}{25} = 1$$

Unit 3 Day #7 Parabolas

Objective: You will use and determine the standard and general forms of the equation of a parabola and graph.

Parabolas - the set of all points that are the same distance from a given point (focus) and a given line (directrix).



$4p$ = **Latus Rectum**

Parabola Formulas:

Standard Form: $(y-k)^2 = 4p(x-h)$ opens left/right
if p is +, opens right
if p is −, opens left

$(x-h)^2 = 4p(y-k)$ opens up/down
if p is +, opens up
if p is −, opens down

Vertex: (h, k)

Focus: a point located inside the parabola, p-units from the vertex.

Directrix: a line on the opposite side of the vertex, p-units from the vertex.

Axis of Symmetry - a line that cuts the parabola in half.

Example #1

Given $y^2 - 2x + 14y = -41$

Find the vertex, focus, directrix, axis of symmetry + sketch.

$\underbrace{y^2 + 14y + \boxed{49}} = +2x - 41 + \boxed{49}$

$(y+7)^2 = 2x + 8$

$(y+7)^2 = 2(x+4)$

Vertex: $(-4, -7)$

$\frac{4p}{4} = \frac{2}{4}$

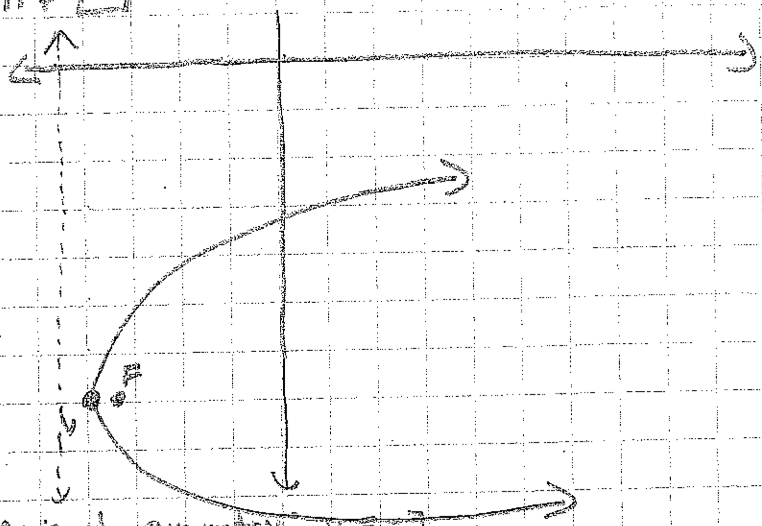
$p = 1/2$ opens right

Focus: $(-4 + 1/2, -7)$

$(-\frac{7}{2}, -7)$

Directrix: $x = -4 - 1/2$

$x = -\frac{9}{2}$



Example #2 Given: $3x^2 - 30y - 18x + 87 = 0$

Find the vertex, focus, directrix, axis of symmetry + sketch.

$3x^2 - 18x = 30y - 87$

$3(x^2 - 6x + \boxed{9}) = 30y - 87 + \boxed{27}$

$\frac{3(x-3)^2}{3} = \frac{30y - 60}{3}$

$(x-3)^2 = 10y - 20$

$(x-3)^2 = 10(y-2)$

Vertex: $(3, 2)$

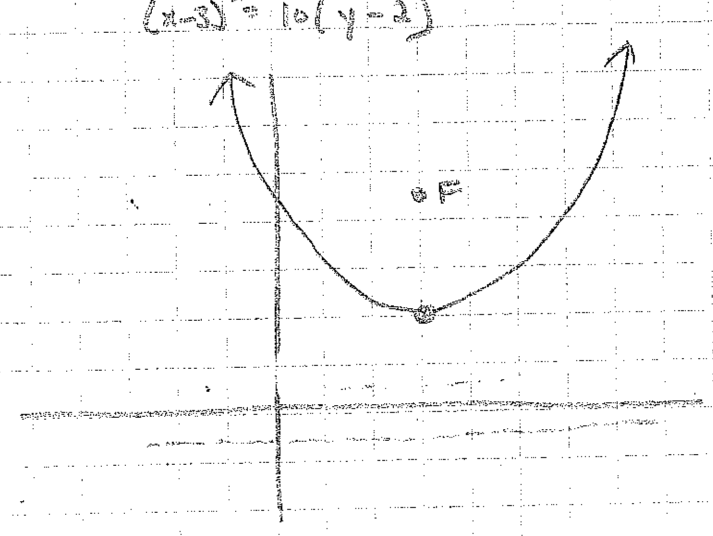
$\frac{4p}{4} = \frac{10}{4}$

$p = \frac{5}{2}$ opens up.

Focus: $(3, 2 + \frac{5}{2}) = (3, \frac{9}{2})$

Directrix: $y = 2 - 5/2 = -1/2$

Axis of symmetry $x = 3$



HW: Find vertex, focus, directrix, axis of symmetry + sketch

1) $y - 2 = x^2 - 4x$

2) $2x^2 - 12y - 16x + 20 = 0$